TWO-SCALE ANALYSIS OF CRACK USING THE EXTENDED FINITE ELEMENT METHOD

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Summary. *The aim of this work is to solve mechanical problems on large structure containing details using a two-scale analysis. The structural scale is solved using a mesh which does not take into account the detail. The detail is taken into account by an "extended" homogenization. The homogenization is obtained from a local analysis of the detail on a sub-mesh of the structural one. On this local mesh, the X-FEM approach is used to carry out the analysis.*

1 INTRODUCTION

The aim of this work is to solve problems on a large structure containing a detail with a two-scale analysis. A coarse mesh is used for the structure, while a fine mesh is superimposed in a local zone surrounding the detail. In this region, the stiffness matrices of the coarse mesh elements are modified from the solution of elementary microscopic problems. Therefore, the microscopic and macroscopic problems are treated successively, which differs from other works in this area[1,2]. Moreover, the microscopic problems are solved using X-FEM to avoid to mesh complex details or to be able to vary easily the detail geometry (for instance, if the crack grows). The contributions of this work are, first, to preserve at the macroscopic scale a standard finite element method (only the stiffness matrix is affected) and second, to use the X-FEM for the microscopic scale.

2 The two-scale discrete problem

The finite element space is denoted by $V^{H,h}(\Omega)$:

$$V^{H,h}(\Omega) = \left\{ \begin{array}{l} \boldsymbol{v} \text{ such that } \boldsymbol{v} \in V^H(\Omega \setminus \omega) \text{ on } \Omega \setminus \omega \\ \boldsymbol{v} \text{ such that } \boldsymbol{v} \in V^h(\omega) \text{ on } \omega \\ \text{and } \boldsymbol{v} \text{ continuous across } \partial\omega \end{array} \right\}$$

where $V^h(\omega)$ on ω, with a mesh size h, is the microscopic space and $V^H(\Omega \setminus \omega)$, with a mesh size H, the macroscopic space. The microscopic mesh is a sub-mesh of a set of macroscopic elements (figure 1). Continuous displacements are imposed on the boundary of ω. The variational formulation is to find $\boldsymbol{u}^{H,h} \in V^{H,h}(\Omega)$ such that:

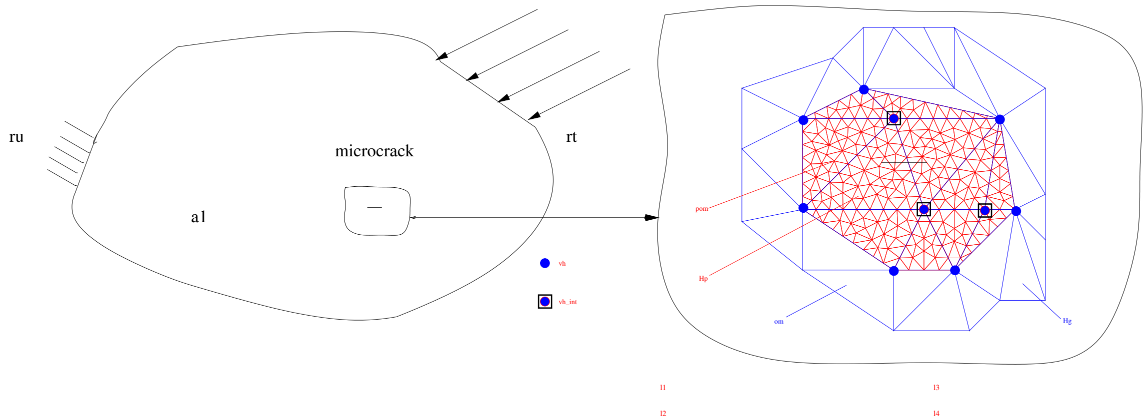

Figure 1: The two-scale problem

$$\int_{\Omega} \boldsymbol{\varepsilon}(\boldsymbol{u}^{H,h}) : \boldsymbol{E} : \boldsymbol{\varepsilon}(\boldsymbol{v}) \; d\Omega = \int_{\Gamma_t} \boldsymbol{F} \cdot \boldsymbol{v} \; ds \quad \forall \boldsymbol{v} \in V^{H,h}(\Omega) \tag{1}$$

The discrete problem is considered as a two-scale problem. The solution ($\boldsymbol{u}^{H,h}$), defined on Ω, is computed like a macroscopic space defined on Ω and improved by a microscopic space ($\boldsymbol{u}^{h,\perp}$) defined on ω.

$$\boldsymbol{u}^{H,h} = \boldsymbol{u}^{H} + \boldsymbol{u}^{h,\perp} \;\; , \boldsymbol{u}^{H} \in V^{H}(\Omega), \;\; \boldsymbol{u}^{h,\perp} \in V_0^{h,\perp}(\omega) \tag{2}$$

with: $V_0^H(\omega) = \left\{ \boldsymbol{v} \in V^H(\omega) \text{ and } \boldsymbol{v} = \boldsymbol{0} \text{ on } \partial\omega \right\}$, $V_0^h(\omega) = \left\{ \boldsymbol{v} \in V^h(\omega) \text{ and } \boldsymbol{v} = \boldsymbol{0} \text{ on } \partial\omega \right\}$ and $V_0^{h,\perp}(\omega) = \left\{ \boldsymbol{v} \in V_0^h(\omega) \text{ and } \int_{\omega} \boldsymbol{v} \cdot \boldsymbol{\lambda}^H \; d\omega = 0 \;\; \forall \boldsymbol{\lambda}^H \in V_0^H(\omega) \right\}$

The problem (1) is solved as find $\boldsymbol{u}^H \in V^H(\Omega)$, $\boldsymbol{u}^h \in V_0^h(\omega)$, $\boldsymbol{\lambda}^H \in V_0^H(\omega)$ such that:

$$\begin{cases} a_{\Omega}(\boldsymbol{u}^H + \boldsymbol{u}^h, \boldsymbol{v}^H) \;\; = \int_{\Gamma} \boldsymbol{F} \cdot \boldsymbol{v}^H \; ds \;\; \forall \boldsymbol{v}^H \in V^H(\Omega) \\ a_{\Omega}(\boldsymbol{u}^H + \boldsymbol{u}^h, \boldsymbol{v}^h) \;\; - \int_{\omega} \boldsymbol{\lambda}^H \cdot \boldsymbol{v}^h \; d\omega = 0 \;\; \forall \boldsymbol{v}^h \in V_0^h(\omega) \\ - \int_{\omega} \boldsymbol{u}^h \cdot \boldsymbol{\mu}^H \; d\omega = 0 \;\; \forall \boldsymbol{\mu}^H \in V_0^H(\omega) \end{cases} \tag{3}$$

where we use the bilinear form : $a_{\Omega}(u, v) = \int_{\Omega} \boldsymbol{\varepsilon}(\boldsymbol{u}) : \boldsymbol{E} : \boldsymbol{\varepsilon}(\boldsymbol{v}) \; d\Omega$.

The system of equation (3) is solved by a two-step procedure.

First, with equations (3.2) and (3.3), we eliminate $\boldsymbol{u}^h$ and $\boldsymbol{\lambda}^H$ from the system by considering u^H as a parameter. The purpose is to obtain elementary solutions as a response to imposed structural loads by activating macroscopic modes.

Then, a macroscopic problem (3.1) for $\boldsymbol{u}^H$ is solved.

The use of lagrange multipliers enable to impose an orthogonal condition between the two displacement fields to avoid a linear relation between each other. The microscopic scale field $\boldsymbol{u}^h$ incorporates the X-FEM enrichment[3].

3 Numerical experiments

A plate with a crack under tension is treated with our approach, and on an overkill mesh taking into account the crack in order to obtain a reference solution (see figure (2.a)).

The figure (2.b) illustrates the microscopic response ($\boldsymbol{u}^{h,\perp}$) of structural load on the fine mesh and the figure (2.c) the total displacement field ($\boldsymbol{u}^{H,h}$) for this same zone. The last two figures compare the results, in term of strain, between our approach and the resolution of the same problem on a uniform mesh of size h used over the entire domain.

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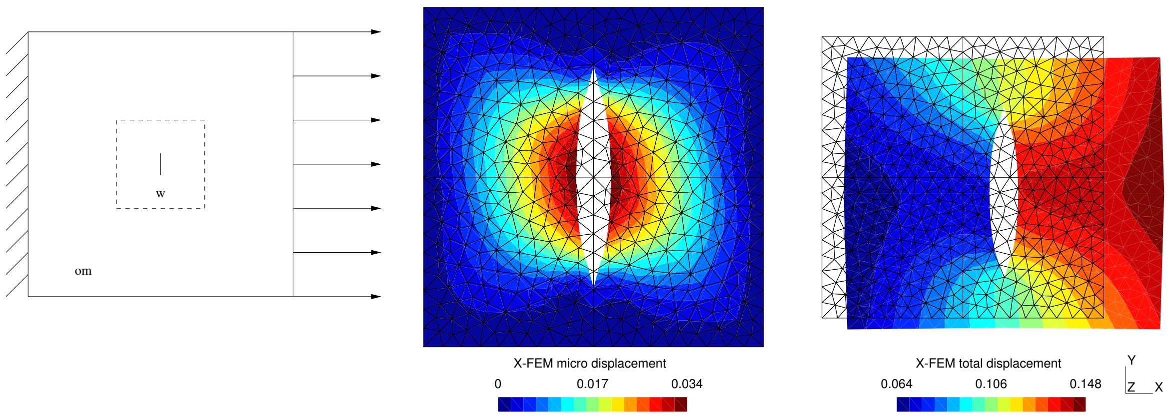
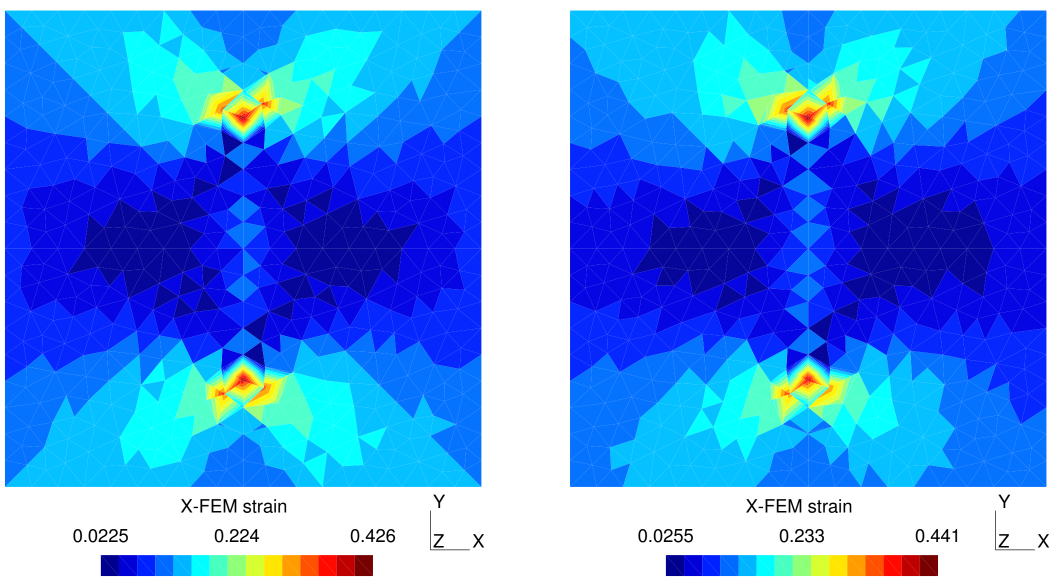


(a) *Plate with a crack under uni-axial tension.* (b) *Micro displacement and micro mesh.* (c) *Total displacement and micro mesh.*



(d) *Total strain for the two-scale approach.* (e) *Strain for the overkill mesh.*

Figure 2: X-FEM results for the plate with a crack